A Network Analyzer Incorporating Two Six-Port Reflectometers

CLETUS A. HOER, MEMBER, IEEE

Abstract—**This paper outlines the theory and design of a microwave network analyzer capable of measuring the network parameters of any linear reciprocal or nonreciprocal, active or passive two port. An RF signal from one source is applied at the same time to two six-port reflectometers which measure the incident and reflected waves at the ports of the two port being measured. An experimental dual six-port network analyzer for the 2–18-GHz range has been completed and is described briefly. Some advantages of the proposed design over existing network analyzer designs are 1) only one source is needed, 2) no phase detectors are required, 3) no flexible cables or arms are used between the reflectometers and the two port being measured, and 4) self-calibration techniques are readily applied.**

I. Introduction

THE SIX-PORT concept has been successfully applied in the realization of microwave reflectometers [1], [2] and vector voltmeters [3]. A conventional six-port reflectometer is a six-port junction designed to measure reflection coefficient and power at one of its ports in terms of power readings taken at four of the other ports when an RF signal is applied at the remaining port. In this paper, two six-port reflectometers are used in the configuration shown in Fig. 1 to measure all of the network parameters of any linear reciprocal or nonreciprocal, active or passive two-port, inserted between the two reflectometers. This configuration is most similar to the interference bridge described in 1962 by Altschuler [4]. The two measurement systems differ in that six-port reflectometers are used in place of slotted lines, isolators, and tuners. This change makes the present system easy to automate and broadband, in the sense that measurements can be made at any fixed frequency from 2–18 GHz after proper calibration at that frequency. Four-port reflectometers could be used in place of six-port reflectometers, but the system would not be as simple as the one described here.

The measurement system described in this paper differs from most other network analyzers in that the six-port reflectometers can be made small enough and portable enough to connect directly to the device under test, eliminating flexible cables or arms between the measurement system and the device being measured. Flexible cables "behind" each six port do not enter into the system calibration except in the measurement of the phase shift through nonreciprocal two ports. The present system is simpler than most other network analyzers in that no IF source is used and no phase detectors are required.

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The author is with the Electromagnetics Division, National Bureau of Standards, Boulder, CO 80302.

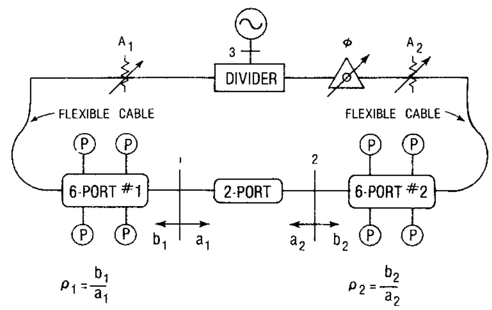

Fig. 1. Basic circuit for measuring all of the scattering parameters of a two port with two six-port reflectometers.

This paper gives the theory behind the measurements, and describes an experimental system which has just been completed. Estimates of the accuracy in measuring attenuation and reflection coefficient are given based on simulated parameters for the measurement system. System accuracy has not yet been experimentally determined.

II. Theory

A. Determine S_{11}, S_{22}, $|S_{12}|$, $|S_{21}|$

The basic setup for using two six-port reflectometers to measure all of the scattering parameters of a two port is shown in Fig. 1. The RF signal is applied to both reflectometers simultaneously. Reflectometer 1 measures the complex ratio $\rho_1 \equiv b_1/a_1$ at reference plane 1. Reflectometer 2 measures $\rho_2 \equiv b_2/a_2$ at reference plane 2. ρ_1 and ρ_2 are not reflection coefficients in the normal sense, but simply relate two traveling waves in opposite directions. These two ratios are related by the S-parameters of the two port inserted between the two reflectometers. Let the scattering equations for the two port under test be

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{1}$$

$$b_2 = S_{21}a_1 + S_{22}a_2. \tag{2}$$

Dividing (1) by a_1 and (2) by a_2 gives

$$\rho_1 = \frac{b_1}{a_1} = S_{11} + S_{12}\frac{a_2}{a_1} \tag{3}$$

$$\rho_2 = \frac{b_2}{a_2} = S_{22} + S_{21}\frac{a_1}{a_2}. \tag{4}$$

Since $|a_2/a_1|$ can be greater or less than one, $|\rho_1|$ and $|\rho_2|$ can also be greater or less than one. Six-port reflectometers are capable of measuring values of $|\rho|$ both larger and smaller than one.

Eliminating a_2/a_1 from (3) and (4) gives

$$(\rho_1 - S_{11})(\rho_2 - S_{22}) = S_{12}S_{21} \tag{5}$$

or

$$\rho_2 S_{11} + \rho_1 S_{22} - \Delta = \rho_1 \rho_2 \tag{6}$$

where Δ is the determinant of the scattering matrix,

$$\Delta \equiv S_{11}S_{22} - S_{12}S_{21}. \tag{7}$$

Three equations like (6) are solved for S_{11}, S_{22}, and Δ. These equations are generated by measuring ρ_1 and ρ_2 for three different values of a_2/a_1 which are determined by the setting of the attenuators A_1, A_2 and of the phase shifter ϕ. The values of A_1, A_2 and ϕ do not need to be known since they do not enter into the equations.

If it is known that the two-port under test is reciprocal, so that $S_{12} = S_{21}$, their amplitudes can be obtained from (7),

$$|S_{12}|^2 = |S_{21}|^2 = |S_{11}S_{22} - \Delta| \tag{8}$$

or from (5),

$$|S_{12}|^2 = |S_{21}|^2 = |(\rho_1 - S_{11})(\rho_2 - S_{22})|. \tag{9}$$

After three or more equations like (6) are solved for S_{11}, S_{22}, and Δ, then S_{11} and S_{22} can be used in (9) to calculate values of $|S_{12}|$ and $|S_{21}|$ for each measurement. The scatter in the values of $|S_{12}|$ gives an indication of the precision with which $|S_{12}|$ has been measured.

If it is not known that the two-port under test is reciprocal, $|S_{12}|$ and $|S_{21}|$ are obtained from (3) and (4) which gives

$$|S_{12}| = |\rho_1 - S_{11}| \left|\frac{a_1}{a_2}\right| \tag{10}$$

$$|S_{21}| = |\rho_2 - S_{22}| \left|\frac{a_2}{a_1}\right|. \tag{11}$$

As will be discussed in Section IV, the six-port reflectometers can be used to measure $|a_2/a_1|$ as well as ρ_1 and ρ_2 in these equations. Altschuler [4] discusses the advantages of making $|a_2/a_1|$ large or small to increase the sensitivity when measuring small values of S_{12} or S_{21}.

B. Determine $\psi_{12} = \psi_{21}$

The equations given in IIA can be used to determine all of the scattering parameters except ψ_{12} and ψ_{21}, the phase angles of S_{12} and S_{21}. For either a reciprocal or nonreciprocal two-port, (3) and (4) give

$$\psi_{12} = \psi_1 - \psi_a \tag{12}$$

$$\psi_{21} = \psi_2 + \psi_a \tag{13}$$

where

$$\psi_1 \equiv \arg\,(\rho_1 - S_{11}) \tag{14}$$

$$\psi_2 \equiv \arg\,(\rho_2 - S_{22}) \tag{15}$$

$$\psi_a \equiv \arg\,(a_2/a_1). \tag{16}$$

Since the six-port reflectometers do not directly measure the phase angle of a_2/a_1, the angle ψ_a is not yet known. The angles ψ_1 and ψ_2 are known.

When the two port is reciprocal, (12) and (13) can be solved for both ψ_a and $\psi_{12} = \psi_{21}$;

$$\psi_{12} = \psi_{21} = \frac{\psi_1 + \psi_2}{2} + n\pi \tag{17}$$

$$\psi_a = \frac{\psi_1 - \psi_2}{2} + n\pi \tag{18}$$

where the ambiguity $n\pi$ arises from the division by 2, and n is an integer. It can be seen from (12) or (13) that n must take on the same value in (17) and (18). The value of n can be determined from either an estimate of ψ_{12} or ψ_a. A good estimate of ψ_a can be obtained from equations derived for the nonreciprocal case which is considered below in Section II-C.

The measurement technique described up to this point is similar to methods which use only one reflectometer to measure ρ_1 when the two port is terminated with three different known terminations at reference plane 2. The equations which apply when using known terminations are the same as (1)–(18) if ρ_2 is replaced by $1/\Gamma_L$ where Γ_L is the reflection coefficient of the termination at reference plane 2. The difference here is that a_2 can be made much larger than if passive terminations are used. This makes it possible to measure larger values of attenuation using the setup in Fig. 1. Another difference is that the ratio $|a_2/a_1|$ cannot be measured when passive terminations are used so that S_{12} and S_{21} of nonreciprocal two ports cannot be measured.

C. Determine $\psi_{12} \neq \psi_{21}$

When the two port under test is not reciprocal, ψ_{12} and ψ_{21} must be calculated from (12) and (13) where ψ_a can be determined as follows. The determination of ψ_a depends in part on knowledge of the measurement system which can be viewed as a three-port junction whose ports are 1, 2, and 3 (near the generator) in Fig. 1. For this three-port junction one can show that

$$\frac{a_2}{a_1} = \left(s_{21} - s_{11}\frac{s_{23}}{s_{13}}\right)\rho_1 - \left(s_{12}\frac{s_{23}}{s_{13}} - s_{22}\right)\rho_2\frac{a_2}{a_1} + \frac{s_{23}}{s_{13}} \tag{19}$$

where the small s_{ij}'s are the scattering parameters of the equivalent three-port measurement system. Rewrite (19) as

$$\frac{a_2}{a_1} = C_1\rho_1 - C_2\left(\rho_2\frac{a_2}{a_1}\right) + C_3 \tag{20}$$

where C_1, C_2, and C_3 are defined by corresponding terms in (19) and (20). The C's for a particular setting of A_1, A_2, and ϕ can be determined from three equations like (20) when ρ_1, ρ_2, and a_2/a_1 are known as discussed below in Section II-D. Solving (20) for a_2/a_1,

$$\frac{a_2}{a_1} = \frac{C_3 + C_1\rho_1}{1 + C_2\rho_2}. \tag{21}$$

Once the C's are known, ψ_a can be calculated from (21) when measuring either reciprocal or nonreciprocal two-ports.

When the two port is nonreciprocal, ψ_a calculated from

(21) is used in (12) and (13) to calculate ψ_{12} and ψ_{21}. When the two port is reciprocal, the value of ψ_a from (21) is considered to be only an estimate for determining n in (18). This is because the value of ψ_a calculated from (18) will, in general, be more accurate than that calculated from (21). Since the C's in (21) are functions of the scattering parameters of the three-port measurement system, they are functions of the stability of the flexible cables and also of the setting and repeatability of the components A_1, A_2, and ϕ. None of the other equations up to (19) are functions of these components or the cables. For this reason, (21) is used only to calculate ψ_a and not $|a_2/a_1|$ which can be obtained from the two six-port reflectometers independently of the properties of the three-port measurement system. Then all of the calculated S-parameters of the two port under test will be independent of the properties of the three-port measurement system except ψ_{12} and ψ_{21} of nonreciprocal two ports.

D. Calibrating the Three Port

The parameters C_1, C_2, and C_3 of the equivalent three-port measurement system need to be determined for use in (21) to find ψ_a. Known values of ρ_1, ρ_2, and a_2/a_1 for calculating three sets of C's corresponding to three different settings of A_1, A_2, and ϕ can be obtained by measuring ρ_1, ρ_2, and $|a_2/a_1|$ for three different reciprocal two ports for which approximate values of ψ_{12} are known. ψ_a will follow from these approximate values. One of these three reciprocal two ports can be a "zero length transmission line," where the two measurement reference planes are directly connected together. The other two reciprocal two ports can be uniform lengths of transmission line of approximately known lengths. Each two port is measured at the same three settings of A_1, A_2, and ϕ. For each of these nine measurements ψ_{12} is calculated from (17), choosing n so that the calculated value of ψ_{12} agrees with the known approximate value. Knowing ψ_{12}, ψ_a is calculated from (12). Then for each setting of A_1, A_2, and ϕ, the calculated values of ψ_a along with the measured values of ρ_1, ρ_2, and $|a_2/a_1|$ are used in (20) to calculate a set of C's for that setting. The C's are then used in (21) to calculate ψ_a when measuring any other two port.

III. Experimental System

A dual six-port automatic network analyzer for use in the 2–18-GHz range has recently been completed. It is outlined in Fig. 2.

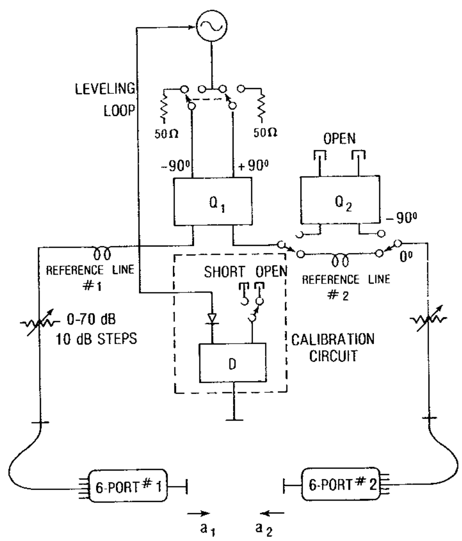

Fig. 2. Dual six-port network analyzer in more detail; Q indicates quadrature hybrid, and D is an in-phase power divider.

A. Network for Changing a_2/a_1

The broadband quadrature hybrids Q_1 and Q_2, and the reference lines 1 and 2, are used to get four different values of ψ_a. Reference line 2 is adjusted so that the difference in phase shift through this line and through Q_2 is constant with frequency. If Q_2 is ideal, this difference in phase is $-90°$. Reference line 1 is adjusted so that ψ_a is also constant with frequency. If Q_1 is ideal, ψ_a will be $+90°$ when the switches are set as shown in Fig. 2. ψ_a can be changed from $+90°$ to $-90°$ by switching the generator to the other input of Q_1. To this phase of $+90°$ or $-90°$ can be added $-90°$ by switching the path from reference line 2 to Q_2. The four possible values of ψ_a are 0°, 90°, 180°, and $-90°$ which are constant with frequency. The actual values of ψ_a can deviate considerably from these ideal values without adversely affecting the performance of the measurement system.

In the present system, the ratios ρ_1, ρ_2, and $|a_2/a_1|$ are measured at all four settings of ψ_a with $|a_2/a_1| \simeq 1$. This results in four equations like (6) instead of the minimum three needed. The four equations are used to get a least-squares determination of S_{11}, S_{22}, and Δ. Then S_{12} and S_{21} are calculated from the appropriate equations. More measurements can be taken at different settings of $|a_2/a_1|$ to increase sensitivity in measuring small values of S_{12} or S_{21}.

B. Calibration Circuit

The calibration circuit in Fig. 2 consists of an in-phase power divider D, a short-circuit and an open-circuit termination, and a diode leveling loop. Its use in calibrating the two reflectometers is discussed in detail in [10].

C. Six-Port Reflectometers

The two six-port reflectometers used are identical in design to the seven-port reflectometer shown in Fig. 3(a), where only four of the five detectors are normally used. Everything above the bottom quadrature hybrid in Fig. 3(a) is the same as the circuit used in the six-port microwave vector voltmeter [3]. Other practical six-port reflectometer designs are described by Engen [5]. Factors to consider in designing a six-port reflectometer are discussed in reference [1]. The value of ρ obtained from a six-port reflectometer is calculated from

$$\rho = \frac{\sum c_i P_i + j \sum s_i P_i}{\sum \alpha_i P_i} \tag{22}$$

which, with few exceptions, applies to any six-port junction [2]. The P_i's are the power readings from four of the five detectors in Fig. 3(a), where P_3 must be one of the detectors used. The 12 real constants c_i, s_i, and α_i characterize the six

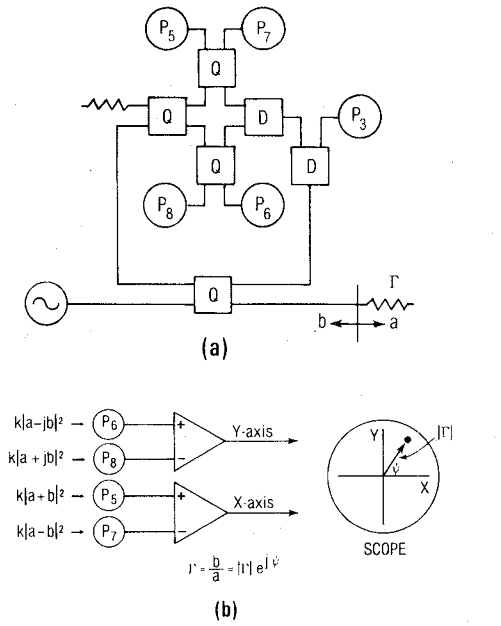

Fig. 3. A seven-port reflectometer constructed with quadrature hybrids Q, and in-phase power dividers D, in a conventional circuit to measure reflection coefficient Γ. A six-port reflectometer is obtained by simply ignoring one of the four outputs $P_5 \cdots P_8$. If the detectors each have an output dc voltage proportional to the input RF power, the voltages can be applied to an oscilloscope as in (b) to get a display of reflection coefficient Γ. The constant k need not be known to display Γ. When the termination is replaced with a two port, the ratio ρ is displayed instead of Γ.

port. They are functions only of the scattering parameters of the six port and of the reflection coefficient of the detectors. They are independent of the properties of the rest of the measurement system including the flexible cables and the generator. One can divide numerator and denominator of (22) by any one of these 12 constants leaving 11 new constants which must be determined by some calibration process as discussed in Section IV.

Using all five detectors, one can display approximate values of ρ on an oscilloscope as shown in Fig. 3(b). This application is based on the following equation for this particular seven-port junction assuming ideal components [2]:

$$\rho = \frac{(P_5 - P_7) + j(P_6 - P_8)}{P_3}. \tag{23}$$

As this equation shows, P_3 acts like a scale factor. P_3 can be measured to determine the scale of $|\rho|$ on the oscilloscope screen. It can also be used to control the level of the generator. A real-time analog display of ρ is especially useful in checking out the measurement system. When reference planes 1 and 2 in Fig. 1 are connected together, a display of ρ_1 is a display of a_2/a_1.

The six-port reflectometers are constructed from stripline in-phase power dividers and quadrature hybrids which cover the 2–18-GHz range. The power detectors are temperature-controlled point-contact diode detectors with video load resistances that have been individually chosen to minimize the deviation from square-law responses [6]. Each

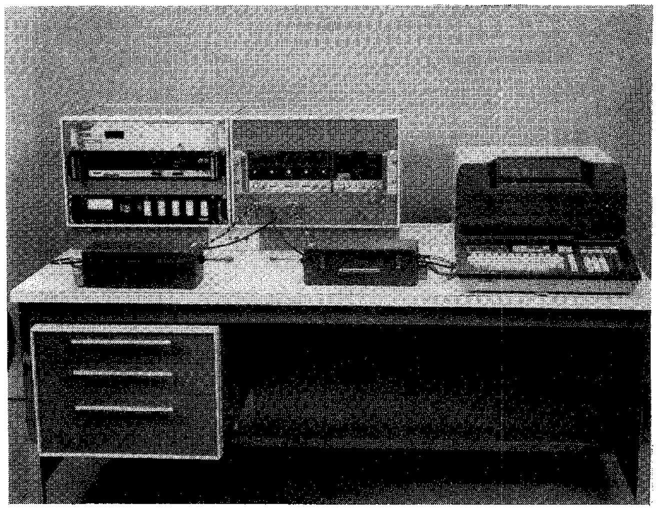

Fig. 4. Photograph of the dual six-port network analyzer.

detector is followed by a low-noise chopper amplifier with a gain of 20 dB. The reflectometers are designed so that either diode detector–amplifier combinations or thermistor mounts can be used. The output voltages of either the amplifiers or the mounts go to a scanner which connects individual outputs to a DVM. The DVM, scanner, source, and all switches are controlled by a programmable calculator which takes the data and performs the calculations to either calibrate the system or find the network parameters of two ports or one ports under test. A photograph of the system is shown in Fig. 4.

IV. Calibrating the Six-Ports

Calibration techniques for determining the constants needed to calculate ρ from equations such as (22) are described in the literature and will only be summarized in this paper. One technique requires the use of two sliding terminations having different nonzero reflection coefficient magnitudes [7]. Another technique requires four different known terminations [8].[1] Either technique yields enough information about each reflectometer to calculate ρ_1 and ρ_2 but not quite enough information to calculate $|a_2/a_1|$. This ratio can be written

$$\frac{|a_2|^2}{|a_1|^2} = \frac{\alpha_{02} \sum \alpha'_{i2} P_{i2}}{\alpha_{01} \sum \alpha'_{i1} P_{i1}} \tag{24}$$

where the second subscript refers to the six-port number. The calibration techniques just mentioned determine the four α'_{i1} and the four α'_{i2} but not α_{01} and α_{02}. The ratio a_2/a_1 can be determined by connecting reference planes 1 and 2 together and measuring ρ_1 and ρ_2. Then

$$\frac{a_2}{a_1} = \rho_1 = \frac{1}{\rho_2}. \tag{25}$$

[1] Since this publication, it has been shown that the technique is valid without assuming that the incident waves to the test loads are constant; however, the solution then becomes an iterative one instead of closed.

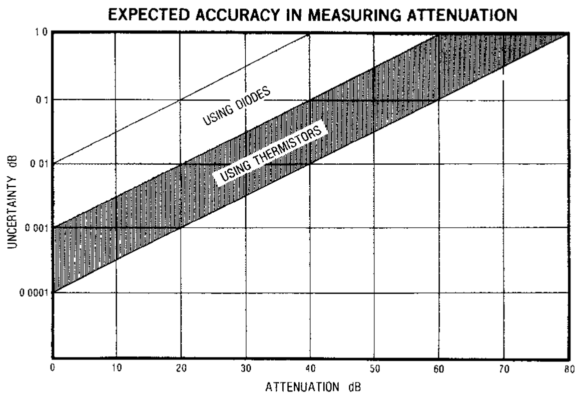

Fig. 5. Expected accuracy ranges in measuring attenuation with the dual six-port network analyzer based on computer simulation of the measurement system assuming imperfect diode or thermistor power detectors.

With $|a_2/a_1|$ known, α_{02}/α_{01} is calculated from (24) where P_{i1} and P_{i2} are the eight power readings used in calculating ρ_1 and ρ_2. Equation (24) can now be used to calculate $|a_2/a_1|$ when any two port to be measured is inserted between reference planes 1 and 2.

Self-calibration techniques similar to those described by Allred and Manney [9] for calibrating two four-port couplers can also be used to calibrate two six-port reflectometers. When applied to the dual six-port measurement system, this technique yields the constants in (22) needed to calculate ρ_1 and ρ_2, and also the constant α_{02}/α_{01} in (24) needed to calculate a_2/a_1. Only one standard is required in the self-calibration technique. This standard can be a uniform length of transmission line or one known termination. Details of this self-calibration technique are given in reference [10]. None of the techniques described in this section determine the C's in (21) needed to calculate ψ_a.

V. Expected Accuracy

Fig. 5 shows the accuracy expected of the system when measuring attenuation. These estimates were obtained by simulating measurement system parameters on the calculator, and letting simulated detector readings take on errors that one might expect as a result of using either diodes or thermistors as detectors. The estimated accuracy in $|S_{11}|$, $|S_{22}|$, or reflection coefficient magnitude is 0.001 to 0.01 for diodes, or 0.0001 to 0.001 for thermistors. Experimental evaluation of the accuracy of the system has not yet been completed.

VI. Summary and Conclusions

Some significant features of the dual six-port automatic network analyzer which are not found in many other measurement systems are:

1) No IF source is required, and the single RF source used need not be a phase-locked source in many applications. No frequency conversion or modulation is used.

2) Only detectors capable of measuring power ratios, that is relative power, are required. Phase measurement capability is not required. Phase is eventually computed from magnitude only data.

3) Measurement accuracy is determined primarily by the linearity and resolution (not dynamic range) of the power detectors. When using thermistor mounts as detectors, accuracy should be comparable to that of the best systems now available.

4) Six-port reflectometers including their power detectors can be made small and portable enough to connect directly to the device under test, thus eliminating flexible cables or arms between the test device and the measurement system.

5) Self-calibration techniques are readily applied so that only one precision impedance standard is required to calibrate the system. This standard can be a uniform length of coaxial line or waveguide which is the most accurate microwave standard available.

6) The dual six-port ANA is relatively easy to automate. All microwave information is obtained from dc voltage measurements.

Many of the above features become even more important as one attempts to measure network parameters above the microwave range. Although the system described uses coaxial components, the theory applies equally well to waveguide systems.

Acknowledgment

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